So far as I can judge from a somewhat cursory examination of works on the spectroscope, it would appear that, when a variable or periodic star is at its maximum of brightness, its spectrum is the ordinary spectrum of the star with certain bright lines, those of hydrogen, added. Now in the stars placed by Secchi in his first class, such as Sirius, the spectrum consists also of a certain assemblage of bright and dark lines, with the characteristic lines of hydrogen superposed.

Art VII.—*Notes upon Mr. Frankland's paper, "On the Simplest Continuous Manifoldness of two Dimensions and of Finite Extent."**

By William Skey, Analyst to the Geological Survey of New Zealand.

[*Read before the Wellington Philosophical Society, 26th June,* 1880.]

It may be still in your mind that, some time ago, one of our members, Mr. F. W. Frankland, read a paper to us embodying a great deal of very remarkable matter, and entitled, "On the simplest continuous manifoldness of two dimensions and of finite extent."* Now, there is much in this paper which I took great exception to at the time, and still do; but I have hitherto refrained from informing you of this, as I had always the hope that a subject in itself so startling and profound, though possibly not new to you, would, as presented to us, and championed in this way, have elicited something more than a mere verbal discussion thereon; something more comprehensive and connected than such a discussion can well be; something commensurate with the importance of the matter treated, and which would possibly represent my ideas thereupon better than I may ever attempt to do.

My hope not being realized I can wait no longer, and I therefore beg your kind attention for a short time, so that I may, as best I can, acquaint you with the particulars of my dissent from the views in question, and my reasons for it; and if, in its turn, this paper should fortunately induce Mr. Frankland to answer the objections which he will here find stated, or to explain those parts of his paper which must appear somewhat obscure to others besides myself, I am sure that, for such a boon, you will cheerfully accord me the time and attention I ask for, and excuse all my shortcomings.

Ere I proceed with this, I will refresh your memory by a synopsis of Mr. Frankland's paper.

It commences by a statement of the well-known fact that some geometricians maintain that the axioms of geometry may be only approximately

* See "Trans. N.Z. Inst.," Vol. IX., p. 272.

true (whatever this may mean), and that, of these geometricians, Lobatchewsky has, by assuming the twelfth axiom of Euclid to be untrue, "worked out the conception of a space in which the ordinary laws of geometry do not hold good." From this and other assumptions at variance with the axioms of Euclid respecting distance relations, it is assumed, as a fact, that geometry is only a particular branch of a more general science, and that "the conception of space is a particular variety of a wider and more general conception." To this wider conception is applied the term "manifoldness," and the full meaning of this term is very lucidly explained.

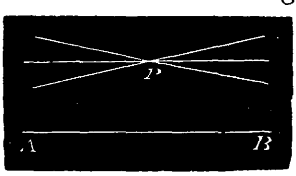
The author then adverts to "the existence" of a particular manifoldness, which has been treated by Professor Clifford in a lecture on the postulates of space; then he describes how this space is analytically conceived, with the object of putting us in a position to apprehend certain discoveries of his own, which relate to its very singular properties; these discoveries communicated to us, he closes his paper with a quotation from Professor Clifford imputing finiteness to the Universe as a result of certain conclusions he has arrived at, which pertain to, or are deducible from, this wider conception, and indicate, on his part, a belief therein, a belief which we may fairly infer is shared in by Mr. Frankland himself.

Such is a fair synopsis of Mr. Frankland's paper, and no one who considers it as a whole, can avoid the conclusion that its prime object is to spread and support the views of the metaphysical school; *i. e.*, it is working for the absorption of science in metaphysics by endeavouring to show that for one of its sections—geometry—there is a transcendental geometry, which not only stultifies it, but swallows it whole, and ultimately assimilates it to itself. This view is supported by the fact, that just recently this gentleman has read before us a very able and profound paper entitled "Mind Stuff," and which is evidently of a highly metaphysical character.

This, however, by the way, has nothing to do with the objections which I here restrict myself to detail and support, and I therefore proceed by premising, that these objections do not extend, at least in a direct manner, to the results of the original researches which are embodied in this paper. Whether these are valid or not is a matter to be only properly tested by examining, as I have, what they rest upon; but this I will say now, (so that the position I take in respect to this matter, may be apprehended at the outset)—that all which this paper treats of, which is distinctly antagonistic to geometry (as understood by geometers of the Euclidian school, or intended to be *conveyed by them*), I take exception to, and now I proceed to show cause. For this I shall, in order to keep myself within due limits, adhere as much as possible to the text of Mr. Frankland's paper, conceiving as I do, and as I have every reason to do, from my knowledge of his mathematical attainments, that the arguments of the geometers he cites therein are correctly rendered and are their best.

To commence then, as to the Euclidian axiom, which Lobatchewsky assumes should be incorrect—namely, the twelfth—that relating to parallel straight lines, an equivalent form of which the author gives as being the one "now generally employed in works on geometry"—it runs thus: "It is impossible to draw more than one straight line parallel to a given straight line through a given point outside it." But, observe, that it is not this equivalent which Lobatchewsky is supposed to use in his attempt at demonstrating the truth of his assumption, but an equivalent of the above-given equivalent; and so, as we have to deal with an equivalent thus twice removed, we must be doubly careful to see whether or not equivalence is here maintained.

As given by Mr. Frankland, this supposed equivalent is as follows: "If we take a fixed straight line, *A B*, prolonged infinitely in both directions,



and a fixed point, *P*, outside it; then, if a second straight line"—(say, *C D*)—"also infinitely prolonged in both directions be made to rotate about *P*, there is *only one* position in which it will not intersect the line *A B*."

Now, I contend that this is not what it purports to be—an equivalent of the Euclidian axiom set before us; and, I think it can plainly be seen that parallelism, as a quality of such lines to be sought for or maintained, is given up; for, allowing the reverse of the proposition to be true—allowing that the second straight line may be made to occupy more than one position relative to the line *A B* without intersecting it, still it will be parallel to it in *one position only*.

In reality the substituted proposition which we have here, does not even touch the original in its essential part; it is, in fact, an independent one, excluding the idea of parallelism as either a necessity or even a desideratum, and merely affirming something which, whether true or otherwise, has nothing to do with the matter now before us.

To any one who will examine the subject, all this must, I think, appear so palpable, that I may leave it now and turn to the discussion of what Lobatchewsky would make of this pseudo-equivalent.

Taking up Mr. Frankland at this point, we have him rendering the master thus: "Now Lobatchewsky made the supposition that this axiom" (meaning, of course, the equivalent in question) "should be untrue, and that there should be a finite angle through which the rotating line might be turned without ever intersecting the fixed straight line *A B*."

Here, then, are two propositions: first, that the lines *A B*, *C D*, though infinitely long, may lay angularly to each other without making an intersection; and, second, that this angularity may be such as to be of a finite value. Now as much is made to turn upon the supposed truth of these propositions it may be expedient (notwithstanding what I have already said) that I should make a few remarks upon this matter also.

You will hardly fail to note the very easy manner in which lines infinitely long are spoken of by the proposition; in effect it says:—Take two such lines, manipulate them in the manner described, and a certain result follows,—just as if this were as sure and tangible an operation, and one not so very dissimilar besides, as that of preparing puddings by a recipe out of some standard cookery book. Surely the mere taking of any line stamps that line as a line of but finite length. However this is, one of these lines is to pass through a point outside the other line, but nothing is said as to the distance away from this line at which the point is to be placed. As we have been started into infinities, it is open to us to place it at an infinite distance away or not; but if, in a friendly spirit towards Lobatchewsky, we place it where his result seems the more likely to be secured, that is at an infinite distance away from *A B*, the proposition becomes simply a truism, and by its wide significance defeats the end desired; for in this way any number of lines may strike through the point *P*, and at all angles to *A B*—each of which may be infinitely extended without intersecting this line. It is seen then that this proposition, as it stands, requires amending by an addition thereto which shall restrict us to the placing of the point *P* at a finite distance away from *A B*. Thus checked, we have only now to ascertain whether or no any line inclining to *A B* may be extended infinitely through *P* as now placed, without intersecting this companion line.

It is very difficult for me to work, or even suppose I am working, with lines of such unwieldy length as these we are set to improvise for our geometrical constructions, but it appears to me that even if the angle of convergence is infinitely small the lines would intersect, but not, of course, at any determinable or conceivable distance. It seems that the completion of the ideal construction begun demands this intersection; and yet, on the other hand, I cannot but allow that to realize an intersection at all is to reduce the lines themselves to finite proportions. Clearly, then, dealing in this way with infinities places us on the "horns of a dilemma."

But that the attempt at operating in this way with infinitely long lines is clearly futile if not absurd, is perhaps better manifested by conceiving, or rather trying to conceive, of the exact converse of the proposition in question.

Suppose then, two straight lines infinitely long, joining each other at an angle infinitely small, and the remarkable consequence follows, that at no conceivable length along these lines would they be apart any conceivable distance; and still the "analytical conception" (to use Mr. Frankland's term) is a valid one, that at some point they widen out to such an extent that a line joining their free ends is infinitely long. But then to our further embarrassment we have in this way upon our hands, or rather upon our minds, a triangle infinitely large, knowing full well the while, that aught which has a shape, must ever finite be.

Thus are we again led to conclusions which are self-contradictory, and we learn thereby that geometry is not likely to be advanced or served by us when we go out of our proper beat to soar in the regions of the infinite.

But whatever may be your views in regard to this aspect of the question, which I have thus so superficially and hastily treated, it is perhaps a fortunate thing for my continued sanity, that for his contention Lobatchewsky does not use arguments based upon the properties of lines which converge at angles infinitely small; possibly seeing, as I think we have, that this gives him nothing, he takes us on to the more solid if less extensive ground of the finite. He enlarges the angle which two non-intersecting infinitely extended straight lines in the same plane may make with each other, to a finite one. None of the evidence of Lobatchewsky in favour of this is given by Mr. Frankland, but simply the bare supposition itself. We cannot, therefore, examine the position fairly to Lobatchewsky, but being unaided by his arguments, I feel it impossible to conceive otherwise than that he is in very palpable error.

It appears to me that at any finite angle of convergence of $C\ D$ to $A\ B$ they will intersect at some determinable part of the line $A\ B$, for a finite angle can only mean an angle of such a size that it can be measured or conceived of, or its value numerically assigned. To hold it to be otherwise is really to hold that an angle finitely large is infinitely small, which either is a contradiction, or these qualifying terms are divested of all meaning. This granted, it then follows as a necessary corollary that there is a point along $A\ B$ which the line P will pass through, and a point, too, capable of being exactly determined.

It appears, then, that here Lobatchewsky, in trying to secure something tangible in support of his idea, has overshot the mark, and so entangled himself and his disciples in a fallacy.

If this is so, can we wonder that, starting in this way, Lobatchewsky gets, as Mr. Frankland says, "very curious results." Triangles, the sum of whose internal angles is less than 180°; triangles which get out at their elbows as they grow, and of the same fraternity as introduced to us further on by their patron; straight lines of such potency that any two of them can bind a space; and lastly, as we shall see, a pseudo-spherical surface combining in itself the utmost simplicity with inconceivable complexity;—all these again, wonderful as they are, sinking to insignificance compared with the grand culminating idea (as more recently developed by this new order of geometricians),—a space of four, five, or even seven dimensions—a space which, to us, I suppose like the seventh heaven of Paul, will ever be both inconceivable and impervious.

And, now, proceeding with our observations on Mr. Frankland's paper, we find an Euclidian axiom thus disproved, and such tremendous conceptions as these projected. Mr. Frankland, under the impression that his enthusiastic belief in this has infected us, or that the arguments given are convincing, essays thus to speak in our behalf: "We see, therefore, that geometry is only a particular branch of a more general science, and that the conception of space is a particular variety of a wider and more general conception." Well, geometry may ultimately be thus subordinated. However, I cannot see that its time has come yet.

But a science so capacious—a science which, to us, is transcendental, at least to the less intellectually advanced of us, requires some mark to distinguish it from that which it has developed from, a mark which shall, if possible, indicate some salient or distinguishing feature of it; consequently this is done. Mr. Frankland says: "This wider conception, of which space and time are particular varieties, it has been proposed to denote by the term manifoldness."

To me this is like "giving to airy nothings a local habitation and a name." But we naturally ask, How comes time to be here conjoined with space under the term *manifoldness?* The idea of time is, to say the least, brought in here very abruptly. The explanation of this term in its application to space and time separately I thank him for, but the infinitely harder task of explaining its application to the two conjointly is left to us.

And now, the overthrow of Euclidian geometry being accomplished, a new kind of geometrical science instituted, and a specific feature of it defined and named, Mr. Frankland introduces us to a surface, which, as he says, Professor Clifford has treated as a surface which is taken by him to be "the simplest continuous manifoldness of two dimensions and of finite extent," or, in plainer and shorter English, the simplest surface of limited extent; and as it is upon a surface of this kind that those discoveries are made which it is a purpose of his paper to disclose, he explains how this surface is got, so that we may place ourselves in a position to intelligently follow him.

The conception of this surface is, in the author's own words, arrived at as follows:—"To obtain the simplest case of such manifoldness [*i.e.*, surface] we must suppose that the point towards which two geodesic lines converge is separated from their starting point not by *half* but by the *entire* length of a geodesic line, or what amounts to the same thing, that it *coincides* with the starting point."* Now *it appears to me* that Mr. Frankland is unduly cautious here, in stating as a supposition that which is a fact; for it is certain that any point which is describing a geodesic line, has for its ultimate converging point that identical position whence it started; indeed, as it travels along, it may properly be considered to converge every part of its road in succession, this, however, in a *subordinate* manner; but that part of a geodesic line which happens to be intersected by another geodesic line, is no more a point of convergence for that line than any other part along it. The idea of two principal converging points to every such line seems a false one. On extending a single line of this kind we are not at all impressed with the idea that it converges to a sort of half-way house on its route; the idea of a convergence there, is only got by simultaneously producing two such lines or more. That a geodesic line, then, converges to its own starting point, admits of no supposition, being a fact; but this is not all that is wanted. Two such lines, as heretofore known, enclose two spaces or surfaces; and, for the purpose these latter-day geometricians have in view, it is necessary that they shall enclose but one. This idea, or rather proposition, is conveyed to us in rather a queer manner, considering what it involves and clashes with, viz., (in the retrospective sentence which follows thus),—"It is true that we are utterly unable to figure to ourselves a surface in which two geodesic lines shall only have one point of intersection, and yet shall enclose space." Geodesic lines, then, proceeding from some common point of a surface, are to diverge somehow from the polar of that point; but, at this part, Mr. Frankland, otherwise so full, lucid, and connected, is singularly curt and, to me at least, hardly intelligible, so that it was not until I got nearly through his paper that I found what he omits to inform us of here,—that he is assuming a uniformly curved surface of immense size.† With this knowledge it is manifest that the analytical conception of two geodesic lines refusing to intersect each other more than

* Surely Mr. Frankland must take a positive delight in tormenting us with paradoxes. He gravely informs us here that the finishing point or goal for a geodesic line in process of construction is to be the length of such line away from the starting point of that line. The two points are to be apart, yet *coincide!*

† "And on this ground it has been argued that the Universe may in reality be of finite extent, and that each of its geodesic lines may return into itself, provided only that its *total magnitude be very great as compared with any magnitude which we can bring under our observation.*"—(Frankland, *l. c.*, p. 278.)

once, and so enclosing but one space, is founded upon Lobatchewsky's conception of what parallel straight lines are capable of. The application of this conception to the case in point is not explained by the author, but, if I understand him here aright, a figure is by this means "analytically constructed"—a cross between an ellipsoid and a sphere, with a strain of something undeterminable; a figure so large that its geodesic lines stand clear of each other at one pole; a figure, in fact, of the same rare genus as the burstable triangles of Lobatchewsky. If this *is* the interpretation of the author, and Lobatchewsky's conception is the groundwork of the structure in question, all I need do, in answer to it, is to refer you back to my criticism on the ingenious method employed by this geometrician to raise this very fertile conception.

But, in doing so, I must insist upon Mr. Frankland adhering to the limitation which Lobatchewsky has imposed or submitted to in respect to the angle at which his geodesic lines are to incline unto each other, that is, it is to be of a finite value; not that this is necessary to insist upon for my argument, but that in a way which is authorized by this geometrician it excludes from consideration here geodesic lines which incline to each other at angles which are infinitely small, a labour which I feel fully persuaded would result in nothing, although it has a promising appearance.

Summing up these results of mine upon the subject of Mr. Frankland's paper, it is now, I think abundantly evident that the analytical conception of a surface such as the one which has been worked upon for the discoveries therein communicated, is not in reality valid, and that though possibly not self-contradictory, as Mr. Frankland urges, it requires premises which *are* of this nature; that, in fine, this conception, and the whole of the assumptions which have been formed upon it, are based upon fallacious reasoning. As a consequence of this, therefore, it remains to us that the simplest surface of finite extent which is even analytically conceivable only, (or as Mr. Frankland puts it, "the simplest continuous manifoldness of two dimensions and of finite extent"), is that of a sphere.

All now which I desire to do further in this matter is to make a few remarks upon the quotation from Professor Clifford's "Postulates of Space," with which Mr. Frankland closes his paper, as not to do this would be to leave unchallenged (that is in a direct manner), the very remarkable conception which it is evident Mr. Frankland has all along been preparing us for—a conception, indeed, which I am fain to consider has a value, but this only in showing to what lengths theories of the kind described lead us when indulged in without stint. Magnificently suggestive as the Professor is here, he is only so by stultifying the Universe to us—defaming it as it were—levelling it down to our own plane. Evidently referring to the idea that the Universe is of finite extent, because, (as Mr. Frankland in effect puts it), the properties of any small area of a sphere do not "sensibly differ" from those of a plane, he argues that "in this case the Universe is again* a valid conception," (by the way a curious sort of equivalence this) "for the extent of space is a finite number of cubic miles."

Observe here the very important qualifying term *sensibly*, which forms a part of the proposition but is omitted in the *deductions*. To make the conclusion agree with the premises, it should have gone no further than to affirm that the Universe may not *sensibly* differ from an infinite one. Unscientific and illogical conclusions of a very startling character are easily got by suppressions of this kind—suppressions which lead us all unconsciously to mistake appearances for realities.

Proceeding, however, with this quotation, we observe further that the Professor, having perchance, after all, some doubts as to the validity of this deduction, or possibly forgetting he has *proved* it, essays to prove it again; he says, "and this (finiteness of the Universe) comes about in a very curious way. If you were to start in any direction whatever, and move in that direction in a perfect straight line, according to the definition of Leibnitz, ... you would arrive at this place. Only if you had started upwards you would appear from below."

Mark, now, the qualification put upon straight lines, "*straight according to Leibnitz*,"—put, no doubt, all in good faith, as explanative of straight lines, it does still, I feel assured, confer upon them properties which straight lines have not, and in such a way as has not manifested itself to him, able as he undoubtedly is. To those geometricians whose grosser ideas forbid their translation to that high realm of thought where this new geometry is analytically conceivable, it *does* seem that a definition of straight lines which allows of the idea being held that a man can get back on his tracks by going straight away from them, is a definition that is *just* a little wrong.

Our idea of what straight lines are is a fixed and definite one, whether or not we can get a diagrammatical or verbal definition of them,† and it is

* Referring, I suppose, to that happy time when the firmament was held to be a solid, studded with sparks, and the earth a plane supported on pillars. Delusions once started seem to be ever perennial, except indeed that they are modified to *suit the times*. I daresay in a few centuries this geometry will give way in its turn to something if possible, still more transcendental, and so *ad infinitum*.

† This so-termed definition is on all sides acknowledged to be no definition at all in a strict sense. Euclid's meaning is *clear*, although the terms used are ambiguous, and do not exactly fit it. But our conceptions of what is the necessary property of parallel lines should not be affected thereby. The fact that the definition when "worked out" lets in lines which were not contemplated by Euclid does not make these parallel, but merely shows the faultiness of this definition.

not to be strained or overturned by a statement which falls short of or overreaches its mark.

That the Professor should experience a sense of "relief" in hugging to himself this dwarfed idea of the Universe is not the least of the many curiosities we have been favoured with. Coming from one appreciative of the utility and beauty of science—from one who has often given it a helping hand, it does seem an anomalous thing that he should thus delight in a conception which narrows its field down from an infinite to a finite extent, so that he can avoid contemplating what he is pleased to style "the dreary infinities of homaloidal space."

Well, tastes differ, and mine accords with a belief which is diametrically opposite to this of Professor Clifford and his disciples, a belief that not only is the Universe infinitely extended, but that its constituents are infinite in kind, infinite in quantity, presenting aspects infinitely diverse to us, according to our standpoint, and in none of these aspects, whether in infinitesimal parts or as a whole, to be conceived of by any finite mind, however discriminating or comprehensive its grasp; a sealed book to all, except by scientific aids, but not wholly to be revealed even by these; an eternal enigma always resolving, yet never to be resolved—a Universe whose laws and phenomena are to be interpreted and discovered in so far as can be, rather by active research than by those mystical constructions which we have just considered, and the criticism of which has been the prime object of this paper.

For my part, I blame making so much in this way of the gap "in the chain of reasoning," by which the truths of geometry should be logically connected and represented; but more I blame this illegitimate fecundity of idea—this ill-directed creative power—which, out of the shortcomings of one of its definitions, and the axiom made to supply its deficiencies, would breed this monster to thus devour all that has preceded it. And I conceive that those who forsake geometry, as now defined and understood, to take up with the new, the transcendental philosophy, are really straining out a gnat to swallow a camel.

ART. VIII.—*On Life.* By W. I. SPENCER.

[Read before the Hawke's Bay Philosophical Institute, 11th October, 1880.]

THE paper which I propose reading this evening was not intended originally to be placed before the members of this Institute. As you are aware, a proposal was made some time ago that I should undertake the direction of a class of biology, in connection with the Athenæum. Owing to one cause and another, however, the opening of this class was delayed until the season was so far