When the mean of the $2n$ angles is obtained the reading-error in the final result is $\pm E_r/\sqrt{2n}$. In the measurement of each angle two sights are taken, and the mean error in each due to sighting is $\pm E_s\sqrt{2}$. The final result as the mean of $2n$ angles is therefore $\pm E_s/\sqrt{n}$.

Combining the errors of reading and sighting gives: Mean error in an angle measured by n reiteration on each face :—

$$r_2 = \pm\sqrt{\left\{\frac{E^2_r}{2n} + \frac{E^2_s}{n}\right\}} \qquad (8)$$

The sighting-error is equal in each method, and if the reading-error is equal to the sighting-error

$$r_1 : r_2 = \sqrt{1 + 2n} : \sqrt{3n}$$

or, for an angle measured by three repetitions and also by three reiterations on each face—

$$r_1 : r_2 = \sqrt{7} : 3$$
$$= 2{\cdot}65 : 3.$$

In the introduction to the "Adjustment of Observations" (2nd edition), by Wright and Hayford, is the following: "The repeating theodolite has fallen far short of the expectations of its first advocates, who hoped that with it the errors of measurement of an angle could be reduced almost indefinitely. The mechanical difficulties have proved insurmountable, and the repeating theodolite is now known to be capable of no greater accuracy than the direction instrument." The writer, after several years' experience in testing the two methods with several instruments, concurs with the above quotation, and is of opinion that preference should be given to the method of directions, which is usually more expeditious in the field.

The amount of eccentricity and the errors of graduations should be determined for every instrument. The analysis is given in most text-books on geodesy and practical astronomy. The following results were obtained by the writer from an 8 in. transit instrument by Troughton and Simms :—

Vertical circle: Verniers apart, 180° 00′ 14·6″; line of no eccentricity, 49° 19′ 00″ (el. face right); correction to vernier, A − 9·3″ (a − 229° 19′). A second analysis of the vertical circle disclosed a small error of 1·51″, due to the pivot being of elliptical form. As the instrument is an old one the wear on the under-surface is quite noticeable.

Horizontal circle: Verniers apart, 180° 00′ 03″; line of no eccentricity, 79° 11′ 30″; correction for a single reading to vernier—A = 4·36″ sin (a − 79° 11′ 30″); B = − 4·36′ sin (a − 79° 11′ 30″). The mean of the two verniers is free from the error due to eccentricity.

In the above formulæ a is the angle or bearing under the vernier. In the case of a traverse running in the direction of 169° or thereabouts, and only one vernier used, the bearing would soon be affected by the eccentricity.

The mean error of one vernier of the horizontal circle of the above instrument due to errors of graduation and accidental errors of reading was found to be ± 2·90″. By taking the mean of the two verniers this is reduced to ± 2·06″.

The sighting can be assessed by the observer. An error in sighting of ± 2″ represent 1½ in. at a distance of five miles, and is about equal to one-half the thickness of the signal-pole, and is probably the maximum displacement of the central wires, in the field of view from the signal, that occurs with even an indifferent observer. Accepting this value will give ± 2·83″ as the mean sighting-error in the measurement of an angle. Combining these results by the formula—

$$r_2 = \pm\sqrt{\left(\frac{E_r^2}{2n} + \frac{E_s^2}{n}\right)}$$

where n is three reiterations on each face, the usual number of observations at each station in minor triangulation.

Substitute these values of E_r and E_s.

$$r_2 = \pm\sqrt{\frac{(2{\cdot}06)^2}{6} + \frac{(2{\cdot}83)^2}{3}}$$

or

$$r_2 = \pm 1{\cdot}89''.$$

The above amount represents the mean error in an angle measured by this 8 in. theodolite.

Using this result to compute the accurracy in the chain of triangles gives—

$$\frac{E_z}{c} = r_2 \times 4 \times \cot 50^\circ = \frac{1}{33000} \text{ (nearly)}.$$

The increased accuracy obtained by using better instruments in a triangulation survey for the angular measurements is obvious from the above result, and is much preferable to increasing the size of the triangles or measuring a greater number of base-lines.

The results obtained are that a 5 in. theodolite can only be expected to give an accuracy of 1 in 9,000 in a minor-triangulation survey, and that an accuracy of 1 in 18,000 is required for controlling the ordinary traverse by 5 in. theodolite and long measuring-tapes. Further, the desired accuracy can be easily obtained or exceeded by adopting 7 in. or 8 in. theodolites for the angular work.

Fuller investigations of the results contained in this article can be found by consulting the following authorities: "Astronomy" (Chavenet); "Geodesy" (Crandal); "Effects of Errors in Surveying" (Briggs); "Progress of Geodesy" (McCaw); "Adjustment of Observations" (Wright and Hayford).