Similarly, if z is the n^{th} line—

$$\frac{E_z}{z} = \pm \sqrt{\left\{ \nu^2 \left(\begin{smallmatrix}\text{sum of squares of cotangents of angles} \\ \text{opposite lines used as bases}\end{smallmatrix} \right) + \left(\frac{E_c}{c} \right)^2 \right\}} \qquad (5)$$

When the triangles are well conditioned, or are nearly equilateral in shape, (3) reduces to—

$$\frac{E_z}{z} = \pm \sqrt{\frac{\nu^2\, 2n}{3} + \left(\frac{E_c}{c} \right)^2}$$

If the n^{th} line returns to the base, and since the error in the base is independent of the angle observations, the total error due to angular measurement is given by—

$$\frac{E_z}{c} + \pm \nu \sqrt{\frac{2n}{3}} \qquad (6)$$

Taking the triangles around the boundary of a district, a fair average of the number of triangles in the chain is 15, and therefore $n = 16$.

If $\nu = 7'' = 10^{-5} \times 3{\cdot}394$ in radians, then $\frac{E_z}{c} = \frac{1}{9000}$, very nearly.

This is the most favourable result that can be expected in this class of survey, since the triangles are all equilateral. If 50° is taken as a fair average for obtaining the angular errors the resulting error due to angular measurement is $\frac{E_z}{c} = \frac{1}{8800}$.

These results are usually obtained by actual experience in minor triangulation, in which the angles are measured with a 5 in. theodolite.

For the triangulation to be of value as a controlling agent the accuracy should be 1 in 18,000 instead of 1 in 9,000, as determined by the error in the summation of the angles of a triangle not exceeding 21″. The accuracy of the work can be increased by reducing the number of triangles, with a corresponding increase in the length of the sides, or by measuring bases at intermediate points, or by reducing the error in the summation of the triangles. The first method, of making the triangles larger, is the most economical, but it suffers from the disadvantage that the stations are widely separated and are often too distant to be available for checking traverses that do not extend from one station to the next. The second method, of measuring base-lines at more frequent intervals, has very little to recommend it. In the first place, suitable base-lines must be situated on fairly level or easy country to measure over, and even on flat plains the bases would require to be very numerous to add greatly to the accuracy of the results. Thus to double the accuracy, or to obtain 1 in 18,000, would require a measured base-line at every third triangle. The third method, of reducing the error in the summation of the angles of the triangles, is the most practicable, but it means the employment of larger and more powerful instruments than 5 in. theodolites.

To find the error in the summation of the angles of the triangles, to ensure an accuracy of 1 in 18,000 between the measured and computed values through a chain of 15 triangles, the average angles from which the (cotangents)² are used being taken as 50°, gives, by using the formulæ (4), 10″; or the mean error in each angle should not exceed $3\frac{1}{3}''$. To obtain this result the methods of observing and the instrument used will require a short notice.

There are two methods in general use for observing angles—namely, repetition and reiteration. In repetition, an angle is multiplied a number of times on the graduated limb; the result is obtained by dividing the total angle by the number of repetitions. In the reiteration or direction method the angle is obtained as the mean of a number of simple measurements on different parts of the graduated circle.

Let ν'' denote the error in the measurement of an angle. Now, ν is a total error due to two principal causes—viz., the error of reading and the error of sighting. Let E_r be the mean error in taking one reading of a vernier or reading-microscope. Now, E_r will include the effects of uneven graduation of the divided circle. Let E_s denote the sighting-error and be held to include the small errors due to imperfect levelling, and instrumental errors not completely eliminated by the act of observing on both faces. Consider an angle measured by n repetitions on each face, by an instrument equipped with two verniers. Each vernier is read twice in obtaining the multiple angle on each face. The mean error of reading of the multiple angle is therefore $\pm E_r \sqrt{2}$ for each vernier. Dividing the multiple angle by n and taking the mean of the two verniers reduces the reading-error for each face to $\pm E_r/n$, or a final result of $\pm E_r/n\sqrt{2}$ for the mean error of reading.

Again $2n$ sights are taken on each face, and their influence on the multiple angle is therefore $\pm E_s \sqrt{2n}$, or $\pm E_s \sqrt{\frac{2}{n}}$ on the quotient, and taking the mean of the two faces $\pm E_s/\sqrt{n}$ is the mean error of sighting.

Combining the errors of reading and sighting gives: Mean error in an angle measured by n repetitions on each face,—

$$\nu_1 = \pm \sqrt{\left\{ \frac{E_r^2}{2n^2} + \frac{E_s^2}{n} \right\}} \qquad (7)$$

When an angle is measured by n reiterations on each face, the mean of $2n$ angles is taken, each angle being measured separately by both verniers. Reading one angle by one vernier a mean reading-error of $\pm E_r\sqrt{2}$ results, which reduces to $\pm E_r$ by taking the mean of both verniers.