lines 4–5, 5–6, and 6–7. The error of closure due to chaining is small compared with the total angular error, the angular error being about two and a half times as great as the chaining-error, a result that shows the superiority of the steel-tape measurements compared with the theodolite observations. The actual closing error in the above survey, ·051, represents 1 in 58,000, or ·14 per mile, or ·10 in latitude and departure per mile.

In standard work connecting the permanent monuments of a city survey the closing error should not exceed this limit; such small closing errors can only be obtained by the use of special apparatus for measuring, and high-grade theodolites for the angular work.

The results obtained by the above theoretical investigations agree very well with the closing errors in the surveys by experienced and competent surveyors; and for larger limits allowable by the regulations there is the disadvantage that carelessness and inaccuracy may be induced in the field-work, and frequently mistakes amounting to 5 links are not detected on account of the closing error being within the prescribed limits.

A summary of the results and a suggested limit for the errors in the three classes of work investigated can now be given:—

(1.) For traverses in open country which is level and undulating the theoretical value is 0·8 per mile for the closing error in latitude and departure, and 54″ for the error in bearing.

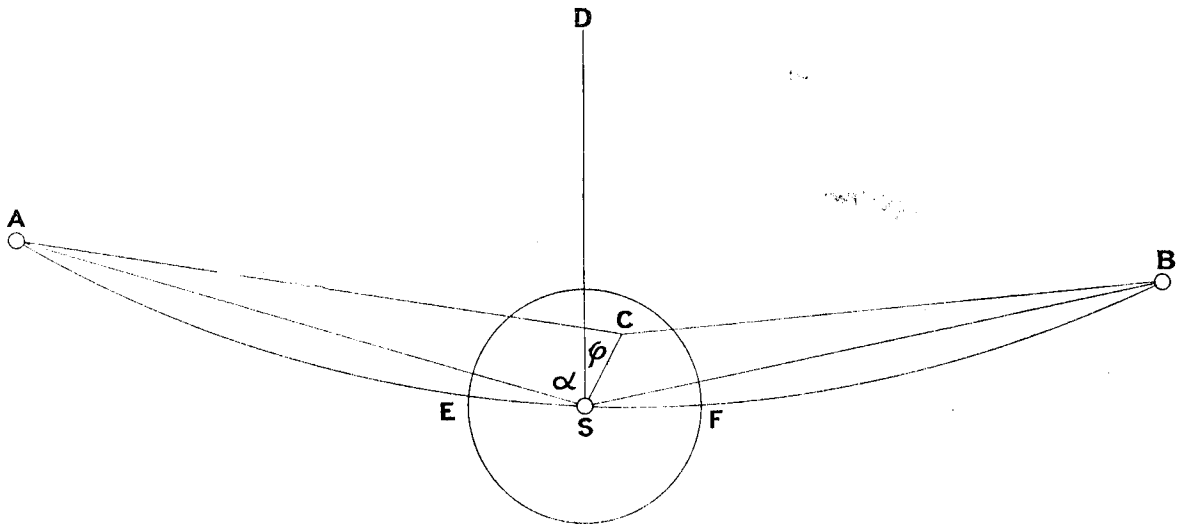
(2.) Traverses in broken forest country or mountainous localities where the lines are short, theory gives 1·9 per mile in latitude and departure, and 2′ 35″ for the closing error in the bearing.

(3.) In town surveys the theoretical values are 0·4 per mile in latitude and departure, and 42″ for the error in bearing.

The suggested limits are as follows: In class (1), 1 link per mile in latitude and departure and 1′ error in the bearing; in class (2), 2 links per mile in latitude and departure and 3′ error in the bearing; in class (3), half a link per mile in latitude and departure and 1′ error in the bearings. These errors are either positive or negative, hence the range of the errors is twice the above amounts.

Error due to Imperfect Centring.

The mean error due to imperfect centring is not given in most text-books on least square. An analytical investigation is given by Briggs in "The Effects of Errors in Surveying," and the following treatment of the problem is somewhat similar:—



Let A, S, and B denote three consecutive stations of the traverse, and r the maximum error permitted in centring. AESB is the arc of a circle passing through the three stations. Describe a circle about S as centre with radius r. Then the point over which the instrument is set will lie within this circle, and the probability of it being over C is equal to that of it being over any other point. Denote the angles ASB by 2α and ACB by 2β. Bisect ASB by the line SD. Denote DSC by ϕ and SC by x. When the instrument stands over C the angle ACB (2β) will be measured instead of ASB (2α), and the angular error is $2(\beta - \alpha)$.

Let AS = D_1 and BS = D_2

$$\sin \text{SAC} = \frac{x \sin (\alpha + \phi)}{D_1}$$

or, since the angle SAC is small,

$$\text{angle SAC} = \frac{x \sin (\alpha + \phi)}{D_1}$$

Similarly, angle SBC $= \dfrac{x \sin (\alpha - \phi)}{D_2}$

$$\therefore \text{SAC} + \text{SBC} = \frac{x \sin (\alpha - \phi)}{D_1} + \frac{x \sin (\alpha + \phi)}{D_2}$$

$$\text{or } 2(\beta - \alpha) = \pm x \left(\frac{\sin (\alpha - \phi)}{D_1} + \frac{\sin (\alpha + \phi)}{D_2} \right) \qquad (14)$$