The problem is to determine the values of R and ζ for any particular tide. If the observations are summed continuously from day $\nu = o$ to day $\nu = n$, then

$$\sum_{\nu=o}^{\nu=n} h_{t,\nu} = R \frac{\sin(n + i\,12{\cdot}5\,i)}{\sin 12{\cdot}5\,i} \cos(i\,t - \zeta + n.\,12{\cdot}5\,i) + \ldots \quad (2)$$

where for the purpose of illustration one term only of (1) is considered, and the sum of the observations from $\nu = n_1 - 1$ to $\nu = n_2$ is the difference between the total sums from $\nu = o$ in each case, or

$$P_t = \sum_{\nu=n_1}^{\nu=n_2} h_{t,\nu} = \sum_{\nu=o}^{\nu=n_2} h_{t,\nu} - \sum_{\nu=o}^{\nu=n_1-1} h_{t,\nu} \quad (3)$$

$$= R \frac{\sin(n_2 - n_1 + 1)\,12{\cdot}5\,i}{\sin 12{\cdot}5\,i} \cos\left\{i\,t - \zeta + (n_1 + n_2)\,12{\cdot}5\,i\right\} + \ldots \quad (4)$$

Put

$$f = \frac{\sin(n_2 - n_1 + 1)\,12{\cdot}5\,i}{\sin 12{\cdot}5\,i} \quad (5)$$

$$N = (n_1 + n_2)\,12{\cdot}5\,i \quad (6)$$

$$\text{then } P_t = R\,f \cos(i\,t - \zeta + N) + \ldots \quad (7)$$

$$= A \cos i\,t + B \sin i\,t + \ldots \quad (8)$$

$$\text{where } A = R\,f \cos(\zeta - N) \quad (9)$$

$$\text{and } B = R\,f \sin(\zeta - N) \quad (10)$$

Now, if the 25 values of P_t are submitted to analysis, we have—

$$\sum_{t=o}^{t=24} P_t \cos 28^\circ{\cdot}8t = F^1 = m\,A + n_1\,B + p\,A_y + q\,B_y + \ldots \quad (11)$$

$$\sum_{t=o}^{t=24} P_t \sin 28^\circ{\cdot}8\,t = G^1 = n_2\,A + r\,B + s\,A_y + t\,B_y + \ldots \quad (12)$$

$$m\,A + n_1\,B = F^1 - (p\,A_y + q\,B_y + \ldots) = F. \quad (13)$$

$$n_2\,A + r\,B = G^1 - (s\,A_y + t\,B_y + \ldots) = G. \quad (14)$$

$$A = \frac{r}{m\,r - n_1 n_2} F - \frac{n_1}{m\,r - n_1 n_2} G. \quad (15)$$

$$B = \frac{m}{m\,r - n_1 n_2} G - \frac{n_2}{m\,r - n_1 n_2} F. \quad (16)$$

$$\text{Cot}(\zeta - N) = \frac{A}{B} \quad (17)$$

$$R = \frac{1}{f}\sqrt{A^2 + B^2} \quad (18)$$

For the M_2 tide, $i = 28^\circ{\cdot}9841042$ per mean solar hour, and the maximum value of f is required. f is a maximum when $\sin(n_2 - n_1 + 1)\,12{\cdot}5i = 1$—that is, when $(n_2 - n_1 + 1)\,12{\cdot}5i = 90^\circ$; hence the maximum value is given by $n_2 - n_1 + 1 = 39$. Hence, if the values of n_2 and $n_1 - 1$ are so chosen as to make $n_2 - n_1 + 1 = 39$, then the corresponding value of f will be a maximum and give the best value of the tide M_2.

As it is desirable to use all the available observations, they are used in continuous sections throughout the year. Thus, beginning with $n_1 = o$, $n_2 = 38$ for first section, then $n_1 = 38$ and $n_2 = 76$ for second section; $n_1 = 76$ and $n_2 = 114$ for third section, and so on.

Schedule for M_2 Tide from List of Sums.

n.	0 h.	1 h.		23 h.	24 h.
38	2275	1906	..	2701	2524
76	3711	3705	..	3792	3737
114	6050	5733	..	6423	6268
..	..	..	..	..	..
304	15001	15008	..	15110	14998
342	17635	17426	..	17667	17664

Example for the third section: $n_2 = 114$, $n_1 = 76$, $n_2 - n_1 + 1 = 39$. Here the sums of the hourly readings are evidently the differences between the total sums opposite $n = 76$ and $n = 114$, and are—

	0 h.	1 h.	2 h.		24 h.	25 h.
P_t =	2439	2028	1675	..	2631	2531

Then $F_1 = + 5061{\cdot}8$, $G_1 = - 7282{\cdot}7$, and for a first approximation the corrections due to the other tides are omitted and F assumed = F^1 and G = G^1. Hence

$$\frac{A}{B} = \frac{+\,425{\cdot}15}{-\,567{\cdot}88} = -\,{\cdot}74866 = \cot 306^\circ{\cdot}82$$

$$\zeta - N = 306^\circ{\cdot}82$$

$$(n_1 + n_2)\,12{\cdot}5i = N = 77\,{\cdot}25$$

$$\therefore \zeta = 24\,{\cdot}07$$

$$A^2 + B^2 = 503233$$

$$\sqrt{A^2 + B^2} = 709{\cdot}39$$

$$\frac{1}{8\,f} = {\cdot}00501935$$

$$R = \frac{\sqrt{A^2 + B^2}}{8\,f} = 3{\cdot}5607 \text{ ft.}$$