

*Algebra.—For Class D, and for Junior Civil Service. Time allowed: 3 hours.*

1. Write down the algebraical expression for the following:—

The number  $x$  is multiplied by itself and is added to the result obtained by multiplying the number  $y$  by itself, and the sum is divided by the sum of the numbers  $x$  and  $y$ ; the quotient is subtracted from that number which when multiplied by itself gives the product of the numbers  $x$  and  $y$ . If  $x=4$  and  $y=9$ , find the numerical value of the final result obtained.

2. Multiply  $3-2x+4x^2-5x^3$  by  $2x^2-x+4$ , and find the value of  $x$  which makes the product exactly divisible by  $x^2-x+3$ .

3. Find the factors of—

$$\begin{aligned} &6a^2-7ab-3b^2; \\ &x^3+a^3-a^2x-ax^2; \\ &2a^2-2b^2-2c^2+3ac-5bc. \end{aligned}$$

4. Simplify  $[1-(1+(1-x))+x^2] \times [-1+(1-(1-x+x^2))]$ ; and find the quotient when  $(x+y)^3+(y+z)^3+(z+x)^3-3(x+y)(y+z)(z+x)$  is divided by  $x+y+z$ .

5. Simplify  $\frac{2x^3-5x^2-4x+3}{3x^3-8x^2-5x+6}$

$$\frac{\frac{a-1}{3} + \frac{a-1}{a-2}}{\frac{a+2}{4} + \frac{a+2}{a-3}} \div \frac{\frac{a+3}{7} - \frac{a+3}{a+4}}{\frac{a-2}{3} + \frac{a-2}{a-1}}$$

6. Show that if  $2s=a+b+c$ , then  $(s-a)^2+(s-b)^2+(s-c)^2+s^2=a^2+b^2+c^2$ , and  $\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} - \frac{1}{s} = \frac{abc}{s(s-a)(s-b)(s-c)}$

7. Solve the equations  $\frac{1}{x-a} - \frac{1}{x-b} = \frac{1}{x+b} - \frac{1}{x+a}$

$$\left. \begin{aligned} \frac{x-y}{2} + \frac{x+y}{3} &= 2\frac{1}{2} \\ \frac{x+y}{2} + \frac{x-y}{3} &= 4\frac{1}{3} \end{aligned} \right\}$$

8. A rectangular field is  $p$  feet long and  $q$  feet wide, and from one corner there is cut off a portion in the shape of a right-angled triangle, of which the right angle coincides with one of the corners of the field, and whose sides, which contain the right angle and which are each  $a$  feet long, are in the direction of the sides of the field. Find the cost of fencing in the remainder at £ $x$  a chain.

9. How much tea at 1s. 8d. a pound must be mixed with 150 lb. at 2s. a pound so that when the mixture is retailed at 2s. a pound a gain of 8 per cent. is made?

10. A cyclist sets out from P and travels at a uniform speed towards Q, which is 180 miles distant. At the same instant as he starts another cyclist starts from Q and travels uniformly towards P. They reach their destinations respectively  $12\frac{1}{2}$  hours and 8 hours after they meet: find the rate at which each travels.

*Algebra.—For Senior Civil Service. Time allowed: 3 hours.*

1. Define the terms *power*, *index*, *cube root*, *reciprocal*.

Show that  $a^0 = 1$ , that  $a^{\frac{1}{3}} = \sqrt[3]{a}$ , and that  $a^{-n}$  is the reciprocal of  $a^n$ .

2. Divide  $x^{\frac{4}{3}} - 2x^{\frac{2}{3}}y^{\frac{2}{3}} + y^{\frac{4}{3}}$  by  $x^{\frac{2}{3}} + 2x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}}$ , and  $25x^{6n} - 16x^{4n} - 8x^{2n} - 1$  by  $5x^{3n} + 6x^{2n} + 2x^n + 1$ .

3. Explain what is meant by the *highest common factor* and the *lowest common multiple* of two quantities, and show that the lowest common multiple is the product of the two quantities divided by their highest common factor.

4. Find the H.C.F. and the L.C.M. of—

$$\begin{aligned} (a.) & 4(x^2y + xy^2); 6(x^3 - xy^2), \text{ and } 9(xy^2 + y^3) \\ (b.) & x^6 + 3x^5 + x + 3 \text{ and } x^3 - 8x + 3 \end{aligned}$$

5. Simplify the expressions—

$$(a.) \quad \left(\frac{1}{a} + \frac{1}{ab^3}\right) \div \left(b - \frac{1}{b}\right)$$

$$(b.) \quad \frac{x - (x^2 - y^2)^{\frac{1}{2}}}{(x+y)^{\frac{1}{2}} - (x-y)^{\frac{1}{2}}} + \frac{x + (x^2 - y^2)^{\frac{1}{2}}}{(x+y)^{\frac{1}{2}} + (x-y)^{\frac{1}{2}}}$$

6. Extract the square roots of—

$$a^{4n+2} + 6a^{3n+1} + 9a^{2n} - 10a^{2n+1}c^{n-2} - 30a^n c^{n-2} + 25c^{2n-4}, \text{ and of } 38 - 12\sqrt{10}.$$

7. Solve the equations—

$$(a.) \quad \frac{1}{x} + \frac{b}{x+a} = \frac{b+1}{x+b}$$

$$(b.) \quad \sqrt{4a+x} = 2\sqrt{b+x} - \sqrt{x}$$

$$(c.) \quad \frac{x+y}{3} + 2z = 21, \quad \frac{y+z}{2} - 3x = -65, \quad \frac{3x+y}{9} - z = 4$$

$$(d.) \quad \frac{a}{x} + \frac{b}{y} = c, \quad \frac{b}{y} + \frac{c}{z} = a, \quad \frac{c}{z} + \frac{a}{x} = b$$