7. If a straight line be divided into two equal parts and also into two unequal parts, the rectangle contained by the unequal parts, together with the square of the line between the points of section, is equal to the square of half the line.

In a right-angled triangle, if a perpendicular be dropped from the right angle upon the hypothenuse, show that the rectangle contained by the sum and difference of the other sides of the triangle is equal to the rectangle contained by the sum and difference of the segments of the hypothenuse.

8. If a straight line be bisected and produced to any point, the square of the whole line thus produced, and the square of the part of it produced, are together double of the square of half the line bisected and of the square of the line made up of the half and the part produced.

If a straight line AB be bisected in C and produced to D, prove that

$$2\,AD.DB = 4\,CD^2 - AD^2 - DB^2.$$

Euclid.—For Senior Civil Service. Time allowed: 3 hours.

1. Prove that if two triangles have two sides of the one equal to two sides of the other, each to each, but the angle contained by the two sides of the one greater than the angle contained by the two sides equal to them of the other, the base of that triangle which has the greater angle is greater than the base of the other.

State how the consideration of three cases is usually avoided in this proposition, and prove that the desired result is attained.

2. Prove that the three bisectors of the angles of a triangle meet in a point.

3. Describe a square upon a given finite straight line.

4. In any triangle show by how much the square on the side subtending an acute angle is less than the sum of the squares on the sides containing that angle.

5. Draw two tangents to a circle from an external point, and prove that they are equal to one another.

6. If from a point without a circle there be drawn two straight lines, one of which cuts the circle and the other meets it, and if the rectangle contained by the whole line which cuts the circle and the part of it without the circle be equal to the square on the line which meets the circle, then the latter line shall touch the circle.

7. Describe a square about a given circle, and prove that all such squares are equal to one another.

Trigonometery.—For Senior Civil Service. Time allowed: 3 hours.

1. Express all the trigonometrical ratios of an angle in terms of the tangent.

If $x \text{ Sin A} = y \text{ Sin B}$, and

$y \text{ Cos A} = x \text{ Cos B}$,

find Tan A and Tan B in terms of x and y.

2. Find the numerical values of Sin 45°, Cot 60°, Cosec 30°.

Find by means of a geometrical construction the cosine and the tangent of an angle of $22\frac{1}{2}°$.

3. Prove the formula $\text{Cos}(\alpha+\beta) = \text{Cos}\,\alpha \text{ Cos}\,\beta - \text{Sin}\,\alpha \text{ Sin}\,\beta$, and deduce the value of Cos 2α.

Give also an independent geometrical proof of the formula you obtain.

4. Prove the relations—

$$\frac{\text{Cos}\,(n-3)\,\text{A} - \text{Cos}\,(n+3)\,\text{A}}{\text{Sin}\,(n-3)\,\text{A} + \text{Sin}\,(n+3)\,\text{A}} = \text{Tan 3A};$$

$$\text{Sin 3A}\,(1-\text{Cot A}) + \text{Cos 3A}\,(1-\text{Tan A}) = 2\,(\text{Sin A} - \text{Cos A}).$$

5. Prove that in any triangle ABC

$$\text{Sin}^2\text{A} = \text{Cos}^2\text{B} + \text{Cos}^2\text{C} + 2 \text{ Cos A Cos B Cos C}$$

$$b\,(\text{tan B} + \text{Tan C}) = a \text{ Tan B Sec C}.$$

6. Show how to solve a triangle when two angles and a side are given.

Given $a = 10$, $\text{A} = 51° 30' 40''$, $\text{B} = 76°$, find b.

Log. 12396 = 4·0932816	L Sin 76° = 9·9869041
Log. 12397 = 4·0933166	L Sin 51° 30′ = 9·8935444
	L Sin 51° 31′ = 9·8936448

7. Find an expression for the area of a triangle in terms of (α) the three sides, (β) one side and the two adjacent angles.

Mechanics.—For Class D, and for Junior and Senior Civil Service. Time allowed: 3 hours.

1. Define *velocity* and *acceleration*, and explain how they are measured. Distinguish between uniform and variable velocity. What is resultant velocity? Show how a velocity may be resolved into horizontal and vertical components.

2. A velocity is represented by 35, and one of its rectangular components is 21: find the other component.

3. A force of 60 poundals acts on a mass of 5 pounds: what is the acceleration? and how far will the mass move in 20 seconds?

4. Define *work*. Explain how work is measured, and in terms of what unit.

What is the work that must be done in order to propel a ball weighing 8lb. at the rate of 100ft. per second?

5. State and prove the "polygon of forces."

6. A uniform rod, weighing 3lb., rests with its ends on two supports, and a weight of 21lb. is hung on the rod at a distance of 2ft. from one end and 3ft. from the other end: find the pressure on the supports.